Integral Relations for Disturbance Isolation

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Consider a system in Fig. 1(a) of two bodies connected with an active strut [1] which is a linear motor. A force disturbance source F_1 is applied to the body M_1 (capital letters designate Laplace transforms). The force F_3 is applied via the massless active strut to the body M_3 To increase the disturbance isolation, the force division ratio $K_F = F_3/F_1$ should be made small, The strut mechanical impedance Z_2 is the ratio of the difference in the velocities at the ends of the strut to the force (since we neglect the strut's mass, the force is same at the both ends of the strut). Feedback is employed in the active strut to increase $|Z_2|$ in order to reduce $|K_F|$.

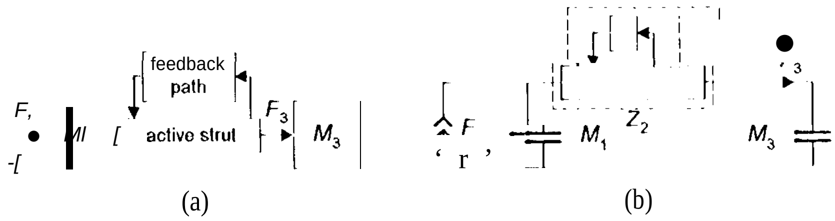

Fig. 1. Mechanical system (a) and electrical equivalent circuit (b)

For the purpose of analysis we use the following electromechanical analogs: power to power, voltage to velocity, current to force, capacitance to mass, and inductance to the inverse of the stiffness coefficient. The electrical equivalent circuit for the system is shown in Fig, 1(b), The current division ratio I_3/I_1 is equivalent to K_F. The electrical impedance Z_2 is the equivalent of the strut mechanical impedance.

The current division ratio, i.e. the force division ratio is

$$K_F = \frac{1/(sM_1)}{1/(sM_1) + Z_2 + 1/(sM_3)}$$

or

$$K_F = \frac{1}{1 + sM_1 Z_2 + M_1/M_3} \tag{1}$$

At higher frequencies, the strut equivalent electrical impedance degenerates into the impedance of the series inductance included in Z_2, the inductance being equivalent to the inverse of the stiffness coefficient k of the strut at higher frequencies. Therefore, the force division ratio at higher frequencies turns into

$$K_F|_{\omega \to \infty} = \frac{k}{s^2 M_1} \tag{2}$$

Since this value reduces as a square of the frequency, Bode integral of the real part of a function [2,3] applies. From this integral,

$$\int_0^\infty \log|K_F + 1| d\omega = 0 \tag{3}$$

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This relation remains valid with and without feedback in the active strut, and allows one to estimate the effect of feedback on the disturbance isolation at higher frequencies.

Another equation which will give a better estimation of the available performance at lower frequencies, can be found as follows, From (1),

$$\frac{1}{K_F(1 + A4_1 / M_3)} = 1 + \frac{sM_1 Z_2}{1 + M_1 / M_3} \tag{4}$$

Consider the practical case of the feedback in the active strut to be negligible at dc. At lower frequencies, the active strut degenerates into some spring. Therefore, at lower frequencies the fraction in the right side of (4) increases with frequency as ω^2. With the frequency scale inverted, the fraction decreases at high frequencies as ω^{-2}. '1'hen, the Bode integral of the real part of a function applies, and from the integral, the integral of the logarithm of the expression in the right side of(4) equals O. Therefore,

$$\int_0^\infty \left[-\log|K_F| - \log(1 + M_1 / M_3)\right] d\omega^{-1} = 0$$

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$$\int_0^\infty \log|K_F| d\omega^{-1} = -\int_0^\infty \log(1 + M_1 / M_3)\, d\omega^{-1} \tag{5}$$

The feedback in the active strut dots not affect the right side of the equation, Therefore, when comparing the cases with different values of feedback in the active strut loops, the right part of (S) can be neglected. Hence, the integral of the difference in the vibration transmission between any two cases with different feedback in the active strut, is zero:

$$\int_0^\infty \Delta \log|K_F| d\omega^{-1} = 0 \tag{6}$$

Equat ion (6) is important because it places a fundamental restriction on what can be achieved by disturbance isolation design, introduction of feedback in the active strut reduces the force division ratio at some frequencies, but at some other frequencies (in fact, al lower frequencies) this ratio increases, and the di fference in the areas of the output force reduct ion and the force increase, with inverse frequency scale, is zero.

1 n experiments with a large-scale model of an interstellar interferometer, a vibration source (representing a reaction wheel) was placed on a platform (body 1) suspended on six orthogonal active struts. Vibration propagation to the base on which sensitive optics was installed (body 2), was *reduced* by the band-pass feedback in the active struts by 30 dB at 20 Hz, the value gradually decreasing with frequency to nearly zero at 100 Hz. At frequency 7 Hz, however, the feedback *increased* the base vibrations by approximately 10 dB. This trade-ofTbct\vccn the vibration amplification and vibration attenuation over different frequency ranges was quite acceptable since al lower frequencies, the feedback in the optical pointing loop was large (sad can be made even larger, if necessary, by application of nonlinear dynamic compensation [3]), and the total error in the optical loops was small.

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References

1. J. Spanos, Z. Rahman, A. von Flotow, Active Vibration Isolation on an Experimental Flexible Structure. Smart Structures and Intelligent Systems Conf., SPIE 1917-60, Albuquerque, 1993.
2. H. W. Bode, *Network Analysis and Feedback Amplifier Design*, Van Nostrand, N.-Y.: 1945.
3. B. J. Lurie, *Feedback Maximization*, Artech House, Dedham, MA: 1986.